Computer Science

MINIMUM VERTEX COVER USING HOPFIELD NETWORKS

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In this paper, we consider a heuristic solution to the minimum vertex cover problem (MVC) using a neural network called a Hopfield network. The MVC problem is an NP-complete type of problem. Thus, this heuristic attempts to solve the problem in polynomial time.

Definition: Given a graph G with vertices V and edges E, a vertex covering Ω for G is a subset of V such that for every edge (u,v) in E, either u or v is an element of Ω. Further, for every vertex covering α, if $|\Omega|\leq|\alpha|$, then Ω is an MVC for G.

A Hopfield network is a complete weighted, symmetric graph. The activation function, $a(h_i)$, for a neuron i is a step function. Therefore, a neuron is either on (has a value of 1) or off (has a value of 0). For $w_{i,j}$, the weight of the connection from neuron i to j, the activation function for neuron i is

$$a(h_i)= 1 \text{ (if } h_i > z\text{), or } 0 \text{ (otherwise)}$$

where $h_i=\sum_j w_{i,j} * a(j)$, is called the total input to neuron i. The total input is summed over every node in the network, and z is the threshold for neuron i.

The first step to solving the MVC problem is to translate a graph G with vertices V and edges E into a neural network. To begin, we create a function $F(x_1 + x_2 + \dots +x_n)$, where $n=|V|$ and each x_i is a Boolean variable. Let F be defined as follows:

$$F(x_1 + x_2 + \dots +x_n)=\prod_{i<j} (x_i + x_j)^c$$

where c=0 if vertices i and j are not connected, and 1 otherwise. Note that only i<j is considered; i connected to j is the same as j connected to i. Next, we find the complement of F. By DeMorgan's law and the fact that x'=(1-x), we get

$$F'(x_1 + x_2 + \dots +x_n)=\sum_{i<j} (1-x_i)(1-x_j)*c$$

After expanding, we can finally create a Hopfield network.

- The number of neurons in the network is |V|
- The weight of each connection is the opposite of the coefficient on the quadratic term in F' (-1 if term is present, 0 if not present)
- The threshold of each neuron is the coefficient of the linear terms

An initial starting value for the activation of every neuron is arbitrarily chosen. The network is then allowed to update itself via the activation functions of each neuron. A stable state is reached when updating any neuron results in no change to the value of its activation. Every neuron that has a value of 1 (i.e. is on) is considered in the MVC of G.

To test the accuracy of the heuristic, 15 graphs were constructed and the method above was applied to find the MVC for each graph. Every possible initial condition was examined for each of the 15 graphs. The average of the percentages of the initial states of each graph that resulted in finding an MVC was approximately 83.333% ± 8.842% with 90% confidence.